Negative refraction via domain wall resonances in a homogeneous mixture of ferro- and nonmagnetic substances

2007 J. Phys.: Condens. Matter 19 072203

(http://iopscience.iop.org/0953-8984/19/7/072203)

IOP FTC
IOP Publishing — Journal of Physics: Condensed Matter

J. Phys.: Condens. Matter **19** (2007) 072203 (7pp) doi:10.1088/0953-8984/19/7/072203

FAST TRACK COMMUNICATION

Negative refraction via domain wall resonances in a homogeneous mixture of ferro- and nonmagnetic substances

Jingsong Wei[1,2] **and Mufei Xiao**[1]

[1] Centro de Ciencias de la Materia Condensada, Universidad Nacional Autónoma de México, Apartado Postal 2681, CP 22800 Ensenada, Baja California, Mexico
[2] Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, People's Republic of China

E-mail: weijingsong@siom.ac.cn and mufei@ccmc.unam.mx

Received 18 November 2006, in final form 9 January 2007
Published 2 February 2007
Online at stacks.iop.org/JPhysCM/19/072203

Abstract
We deliver the general conditions on the synthetic proportions for a homogeneous mixture of ferro- and nonmagnetic substances to become left-handed. As an alternative for left-handed metamaterials, we consider mixing ferromagnetic materials with nonmagnetic microscopic particles. In the mixture, the ferromagnetic material provides the needed permeability via domain wall resonances at high frequencies, whereas the nonmagnetic material gives the required permittivity. Using the effective medium theory, we have found that when the concentration of the nonmagnetic particles falls into a certain range, the refractive index of the mixture is negative, $n < 0$, which includes the double negative ($\epsilon < 0$ and $\mu < 0$) and other cases (e.g. $\epsilon < 0$ and $\mu > 0$). We finally give the requirements on the microscopic material properties for the ferromagnetic materials to reach the domain wall resonances at high frequencies.

(Some figures in this article are in colour only in the electronic version)

In recent times, research activities on the so-called left-handed materials have intensified [1–3] after a number of experimental demonstrations of the property of negative refraction with artificially fabricated metamaterials [4–8].

The general character in a left-handed material is that the relative electrical permittivity ϵ and magnetic permeability μ are simultaneously negative [1–3]. The immediate consequence of this characteristic is that the refractive index n becomes negative, as do the phase velocity v_p as well as the wavevector k for a travelling electromagnetic wave for which the energy flow, as indicated by the Poynting vector $\vec{S}$, remains positive. Since the phase velocity and the energy flow go in the opposite directions, materials with negative refractive index are also called left-handed materials [1].

Due to the left-handedness (and thus the negative refractive index), the new materials are able to generate a few peculiar phenomena, such as the reverse Doppler shift and Cerenkov radiation and the negative refractive angle according to Snell's law [1–3, 9]. Among others, the most researched property so far is the negative refractive angle. In particular, negative refraction has been taken as the fingerprint of left-handed material in various experiments and computer simulations [4–8, 10]. In addition, negative refraction enables light focusing to be realizable using a flat plate [1]. Furthermore, it has recently been claimed that this focusing can be perfect, as the evanescent fields may be amplified by the left-handed plate so that the diffraction limit associated only with the propagating waves would be overcome [2].

Unfortunately, no left-handed media has ever been found in materials in nature, which has resulted in a long silence on this subject for several decades. The main difficulty is that, while most metallic and semiconductor materials do have negative permittivity below the plasmon frequency, materials with negative permeability are rare. Therefore, researchers have resorted to artificial metamaterials [11–14]. The idea is to generate a resonant magnetic response by means of periodic microscopic resonators [2]. At the resonance, one side of the spectral peak may show significant negative permeability. If the $\mu_r < 0$ area is coincident with the $\epsilon_r < 0$ area, within this frequency range the material becomes left-handed. So far, left-handed metamaterials have been realized in the microwave range. For the optical range, the fabrication of metamaterials becomes more complicated and difficult.

The left-handed metamaterials have a few shortcomings. Among others, metamaterials cannot be mass produced and are restricted from high frequencies. Therefore, researchers have been looking for alternatives, primarily a mixture of different substances. For examples, it was recently proposed to make use of metallic magnetic composites under external magnetic field [15], ferromagnetic superconductor superlattices [16], arrays of Josephson conjunctions [17], and nonmagnetic two-component media [18], etc.

In the present communication, we report our finding that in a homogeneous mixture of ferromagnetic materials with microscopic nonmagnetic particles, the refraction can be negative. In the mixture, the ferromagnetic material provides the needed permeability via the domain wall resonances, whereas the nonmagnetic material gives the required permittivity. Using effective medium theory, we have found that when the concentration of the nonmagnetic particles falls into a certain range, the refractive index of the mixture is negative, $n < 0$, which includes the double negative ($\epsilon < 0$ and $\mu < 0$) case for both passive components and other cases for lossy materials [19] and more cases of lossy and gain materials (e.g. $\epsilon < 0$ and $\mu > 0$ etc) [20]. For the domain wall resonances, we have delivered the requirements on the microscopic properties of the ferromagnetic materials.

In looking for the possible negative permeability in ferromagnetic materials, one pays attention to the resonances in the magnetic responses at high frequencies. The possible resonance structures in the permeability stem from the resonance of domain wall motion and the natural resonance. Only the resonance of domain wall motion may show negative susceptibility ($\chi < 0$) and permeability ($\mu < 0$). In ferromagnetic materials, there are lots of magnetic domains with different magnetic moments. The magnetic domains are separated by the domain walls. When the materials are doped with nanometric nonmagnetic particles, the particles are distributed along the domain wall. The domain walls are mobile and may vibrate near the equilibrium positions. If the frequency of the external magnetic field is equal to the eigen-frequency of the domain wall motion, then the resonance occurs, which is called the domain wall resonance.

The domain wall motion is an intrinsic property of magnetic materials. The motion involves the following main energy channels. The static magnetic energy decreases after a distance z as $\Delta E_{\text{static}} = -2\mu_0 M_s H_e z$, where M_s is the saturation magnetization intensity, H_e the external AC magnetic field, and μ_0 the permeability in vacuum. The domain wall energy increase is related to the separation a of the impurities on the domain wall by $\Delta E_{\text{wall}} = \frac{1}{2}\kappa z^2$ with $\kappa = \pi\sigma_{\text{wall}}/a^2$, where σ_{wall} is the unit area domain wall energy. The demagnetization energy induced by domain wall motion is $E_{\text{demagnet}} = \frac{1}{2}m_w(\mathrm{d}z/\mathrm{d}t)^2$ with $m_w = \mu_0 a_0 \sigma_{\text{wall}}/(2\gamma_0^2 A s^2)$, where m_w is the effective mass of unit area domain wall, a_0 is the lattice constant of the cubic crystal, s is the spin quantum number, γ_0 is the gyromagnetic ratio, and A is the exchange integral constant among the close lattice dots. The eddy current loss is related to the electrical resistivity ρ as $P = \beta(\mathrm{d}z/\mathrm{d}t)^2$ with $\beta = 8\mu_0^2 M_s^2/(9\rho)$.

The domain wall motion can be considered to be equivalent to the motion of a body with mass m_w hanging on a spring of restoring coefficient κ. The body is subject to a friction force $\beta\mathrm{d}z/\mathrm{d}t$ and a conservative force $2\mu_0 M_s H_e$. Applying Newton's second law to the domain wall motion, one immediately obtains the following inhomogeneous differential equation:

$$m_w \frac{\mathrm{d}^2 z}{\mathrm{d}t^2} + \kappa z + \beta \frac{\mathrm{d}z}{\mathrm{d}t} = 2\mu_0 M_s H_e. \tag{1}$$

With the harmonic driving field $H_e = H_m e^{i\omega t}$, one solves the equation to get the displacement

$$z = \frac{2\mu_0 M_s}{\kappa} \frac{H_e}{1 - \frac{m_w}{\kappa}\omega^2 + i\omega\frac{\beta}{\kappa}}. \tag{2}$$

Considering the change of magnetization intensity induced by the domain wall motion for a cubic domain of size l as $\Delta M = 2M_s z/l$, and the relation between the susceptibility and the field $\Delta M = \chi\mu_0 H_e$, one finally has

$$\chi = \frac{\chi_0}{1 - \left(\frac{\omega}{\omega_r}\right)^2 + i\frac{\omega}{\omega_\tau}}, \tag{3}$$

with the following definitions: static ($\omega = 0$) susceptibility $\chi_0 = 4M_s^2/(\kappa l) = 4M_s^2 a^2/(\pi\sigma_{\text{wall}} l)$, the resonance frequency of the domain wall motion $\omega_r = \sqrt{\kappa/m_w} = \sqrt{2\pi A s^2 \gamma_0^2/(\mu_0 a_0 a^2)}$, and the relaxation frequency of the domain wall motion $\omega_\tau = \kappa/\beta = 9\pi\rho\sigma_{\text{wall}}/(8\mu_0^2 a^2 M_s^2)$. In figure 1, a typical resonance structure in the susceptibility is shown with both the real and imaginary curves. It may be seen that in the right-hand side of the resonance, the real curve has a negative range. One goes on to obtain the magnetic permeability via $\mu = 1 + \chi$ as

$$\mu = 1 + \frac{\chi_0}{1 - \left(\frac{\omega}{\omega_r}\right)^2 + i\frac{\omega}{\omega_\tau}}. \tag{4}$$

According to the effective medium theory, one gets the generalized permeability from the two components, $\mu_1 = \mu_1' + i\mu_1''$ (the permeability of the ferromagnetic material) and $\mu_2 = \mu_2' + i\mu_2''$ (the permeability of the doped nonmagnetic particles) as

$$\mu = \mu' + i\mu'' = \mu_1 \frac{2\mu_1 + \mu_2 - 2p(\mu_1 - \mu_2)}{2\mu_1 + \mu_2 + p(\mu_1 - \mu_2)}, \tag{5}$$

which defines the relation between the permeability μ and the nonmagnetic material concentration p. In order to have a negative permeability, i.e. $\mu' < 0$, from equation (5) one further gets the following condition:

$$\frac{[p - \mu_1'(2+p) - 1][2(1-p)(\mu_1'^2 - \mu_1''^2) + \mu_1'(1+2p)]}{\mu_1''^2(2+p)[4\mu_1'(1-p) + 1 + 2p]} < 1. \tag{6}$$

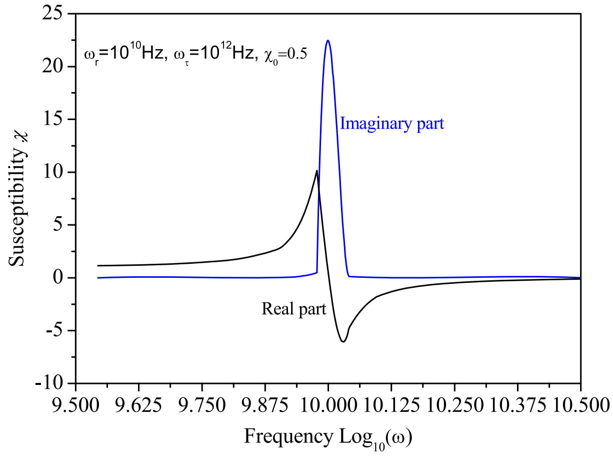

Figure 1. Complex presentations for the real and imaginary components in χ as a function of the frequency.

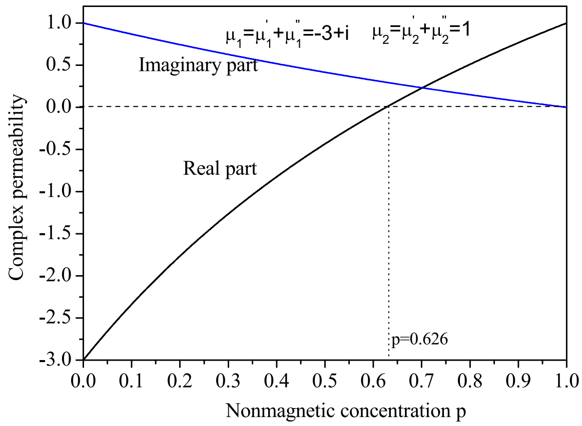

Figure 2. Complex presentations for the real and imaginary components in μ as a function of the nonmagnetic concentration p.

The condition in equation (6) is for the real part of the permeability to become negative and is obtained with the assumption that the nonmagnetic material has $\mu_2 = \mu_2' + \mathrm{i}\mu_2'' = 1$. A numerical example is presented in figure 2 with the parameter $\mu_1 = \mu_1' + \mathrm{i}\mu_1'' = -3 + \mathrm{i}$, where it is shown that the low concentration starting from $p < 0.626$ can have negative permeability $\mu' < 0$.

Similarly, one can apply the effective medium theory to obtain a generalized permittivity ϵ from the two components $\epsilon_1 = \epsilon_1' + \mathrm{i}\epsilon_1''$ and $\epsilon_2 = \epsilon_2' + \mathrm{i}\epsilon_2''$, i.e.

$$\epsilon = \epsilon' + \mathrm{i}\epsilon'' = \epsilon_1 \frac{2\epsilon_1 + \epsilon_2 - 2p(\epsilon_1 - \epsilon_2)}{2\epsilon_1 + \epsilon_2 + p(\epsilon_1 - \epsilon_2)}, \tag{7}$$

which defines the relation between the permittivity ϵ and the particle concentration p. In order to have a negative permittivity, i.e. $\epsilon' < 0$, one can readily get a condition similar to equation (6). However, for a ferromagnetic medium such as ferrite, the material is generally

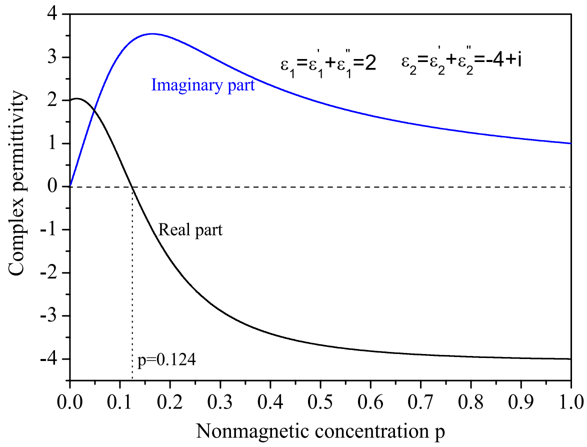

Figure 3. Complex presentations for the real and imaginary components in ϵ as a function of the nonmagnetic concentration p.

also a dielectric medium $\epsilon_1' > 0$ and $\epsilon_1'' \approx 0$. Considering the extra limits, one finally has

$$\frac{[2p(\epsilon_1' - \epsilon_2') - (2\epsilon_1' + \epsilon_2')][p(\epsilon_1' - \epsilon_2') + (2\epsilon_1' + \epsilon_2')]}{\epsilon_2''(1-p)(1+2p)} < 1. \tag{8}$$

In figure 3 we give also a numerical example for the electric permittivity with parameters $\epsilon_1 = 2$ and $\epsilon_2 = -4 + \mathrm{i}$. One realizes that the permittivity does go negative for higher concentration $p > 0.124$.

So far, we have obtained two conditions in equations (6) and (8) for $\mu' < 0$ and $\epsilon' < 0$ simultaneously, and both conditions are related to the concentration p. In order to have a general correlation between the expressions in equtions (5) and (7), one calculates the refractive index. Rewriting the expressions as $\epsilon = |\epsilon| \exp[\mathrm{i}(\alpha + 2m\pi)]$ for $m = 0, 1, 2, \ldots$ and $\mu = |\mu| \exp[\mathrm{i}(\beta + 2m'\pi)]$ for $m' = 0, 1, 2, \ldots$, one eventually obtains the generalized refractive index as $\widetilde{n} = n + \mathrm{i}k = \sqrt{|\epsilon||\mu|} \exp(\frac{\alpha+\beta}{2} + (m + m')\pi)$. Since the calculation includes both positive and negative indices, one verifies the criterion $\mathrm{Re}\{Z\} = \mathrm{Re}\{\sqrt{|\mu|/|\epsilon|}\} \exp[\mathrm{i}(\frac{\alpha-\beta}{2} + (m' - m)\pi)] > 0$ as a guide for the corresponding sign for the index [20, 21]. It follows immediately that, for notation $\widetilde{n} = n + \mathrm{i}k$, the real and imaginary parts of the refractive index can be calculated for both positive and negative cases,

$$n = \sqrt{|\epsilon||\mu|} \cos \frac{\alpha+\beta}{2} \qquad \text{and} \qquad k = \sqrt{|\epsilon||\mu|} \sin \frac{\alpha+\beta}{2}, \tag{9}$$

with the restrictions on the two angles $0 < \alpha < \pi$ and $0 < \beta < \pi$.

Let us now perform a sample calculation for the expressions defined by equation (9) with the following parameters: $\mu_1 = -3 + \mathrm{i}$, $\mu_2 = 1$, $\epsilon_1 = 2$, and $\epsilon_2 = -4 + \mathrm{i}$. The calculated results are presented in figure 4. Figure 4 shows the refractive index as a function of concentration p. One can see that, for $n < 0$, the concentration has to be $0.019 < p < 0.792$. However, within this range, only the central part $0.124 < p < 0.626$ satisfies the so-called double negative case, $\epsilon' < 0$ and $\mu' < 0$. The smaller range $0.019 < p < 0.124$ corresponds to negative refraction under $\epsilon' > 0$ and $\mu' < 0$, while the range $0.626 < p < 0.792$ corresponds the negative refraction under $\epsilon' < 0$ and $\mu' > 0$. These extended ranges for negative refraction have been discussed elsewhere [19, 20]. Even within the central range of the double negative case $0.124 < p < 0.626$, there are areas where the imaginary part is larger than the real part, which may cause confusion for the pure effects of refraction. In order to avoid a strong

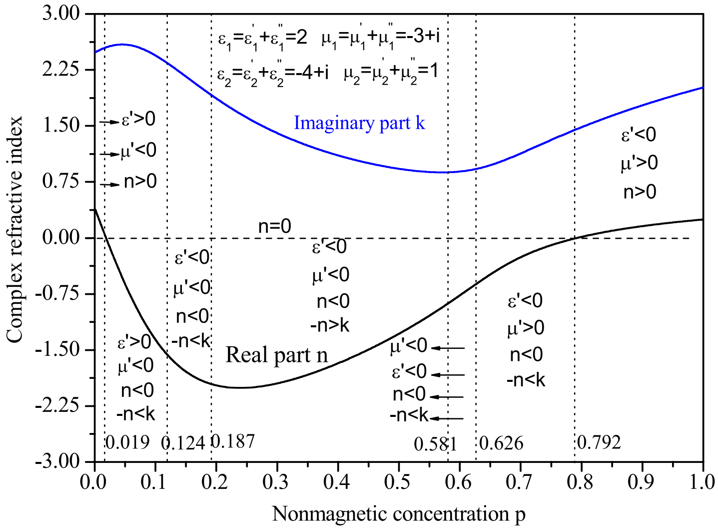

Figure 4. Complex presentations for the n and k in $\tilde{n}$ as a function of the nonmagnetic concentration p.

imaginary part, the concentration should be within a narrower range, $0.187 < p < 0.581$. This is the range where the refractive index is negative, while the imaginary part stays relatively small, $|n| > k$.

The above analysis demonstrates a general route for mixing two materials, for instance one $\mu < 0$ and the other $\epsilon < 0$ at the microscopic level, so that the composite material has a negative refractive index providing that the synthetic proportion falls in a specific range. It is well known that metallic medium has a frequency range for $\epsilon' < 0$. A ferromagnetic medium also has a range for $\mu' < 0$ at high frequencies [22][3]. This negative range stems from the domain wall resonance. In what follows, we further analyse the material properties of ferromagnetic materials to find the appropriate requirements for $\mu' < 0$. We first look for the minimum susceptibility $\chi'_{\min}$. From the expression in equation (3), one realizes that the minimum real susceptibility occurs when $\omega_t/\omega_r \ll 1$ and $\omega \approx \omega_r(1 + \omega_r/2\omega_\tau)$. In this approximation, one obtains $\chi'_{\min} \approx -\frac{1}{2}\chi_0\omega_\tau/\omega_r$. Therefore, for the real permeability to be negative $\mu' < 0$, the following condition holds:

$$\mu'_{\min} \approx 1 - \frac{1}{2}\chi_0\frac{\omega_\tau}{\omega_r} < 0 \Rightarrow \chi_0 > 2\frac{\omega_r}{\omega_\tau}. \tag{10}$$

If we replace the variables in the above expression with microscopic material parameters, we further have

$$\frac{\sqrt{A}\gamma_0 sl}{\rho\sqrt{a_0}a} < \frac{9\mu_0^{-3/2}}{4\sqrt{2\pi}}. \tag{11}$$

Therefore it is generally preferable to choose a material with large resistivity, while the size of the domain should be small. As we have shown in figure 4, there are extended possibilities for producing negative refraction for $\mu' > 0$. In addition, in order to further increase the resonance frequency ω_r, one has to increase the resistivity of the material ρ and meanwhile reduce the saturation magnetization intensity, M_s.

[3] See, for example, that negative permeability at room temperature was shown for $Ba_3Co_2Fe_{24}O_{41}$ at >1 GHz in [23].

In conclusion, we have studied the possibility of obtaining materials with negative refraction by means of mixing ferromagnetic materials with nonmagnetic microscopic particles. We have found that, when the concentration of the metallic particles falls into a certain range, the refractive index of the mixture is negative, $n < 0$, which includes the double negative ($\epsilon < 0$ and $\mu < 0$) and other cases (e.g. $\epsilon < 0$ and $\mu > 0$). We finally give the requirements on the microscopic material properties for the ferromagnetic materials to reach the domain wall resonances at high frequencies.

This work is partially supported by the Shanghai Rising Star Program (No. 05QMX1476), Shanghai Key Basic Research Project (No. 06DJ14007), National Natural Science Foundation of China (No. 60507009 and 60490290), Chinese Academy of Sciences (No. KJCXZ. YW. NANO. 06) and Universidad Nacional Autónoma de México UNAM-DGAPA IN120406-3.

References

[1] Veselago V G 1968 *Sov. Phys.—Usp.* **10** 509
[2] Pendry J B 2000 *Phys. Rev. Lett.* **85** 3966
[3] Pendry J B and Smith D R 2000 *Phys. Today* **53** (5) 17
[4] Shelby R A, Smith D R and Schultz S 2001 *Science* **292** 77
[5] Smith D R *et al* 2002 *Phys. Rev.* B **65** 195104
[6] Parazzoli C G *et al* 2003 *Phys. Rev. Lett.* **90** 107401
[7] Houck A A, Brock J B and Chuang I L 2003 *Phys. Rev. Lett.* **90** 137401
[8] Loschialpo P F *et al* 2003 *Phys. Rev.* E **67** 025602
[9] Seddon N and Bearpark T 2003 *Science* **302** 1537
[10] Foteinopoulou S, Economou E N and Soukoulis C M 2003 *Phys. Rev. Lett.* **90** 107402
[11] Pendry J B *et al* 1996 *Phys. Rev. Lett.* **76** 4773
[12] Smith D R and Pendry J B 2006 *J. Opt. Soc. Am.* B **23** 391
[13] Bayindir M *et al* 2002 *Appl. Phys. Lett.* **81** 120
[14] Markos P and Soukoulis C M 2002 *Phys. Rev.* E **65** 036622
[15] Chui S T and Hu L 2002 *Phys. Rev.* B **65** 144407
[16] Pimenov A, Loidl A, Przyslupski P and Dabrowski B 2005 *Phys. Rev. Lett.* **95** 247009
[17] Du C, Chen H and Li S 2006 *Phys. Rev.* B **74** 113105
[18] Chen Y, Fischer P and Wise F W 2005 *Phys. Rev. Lett.* **95** 067402
[19] Depine R A and Lakhtakia A 2004 *Microw. Opt. Technol. Lett.* **41** 315
[20] Wei J and Xiao M 2007 *Opt. Commun.* **270** 455
[21] Ziolkowski R W and Heyman E 2001 *Phys. Rev.* E **64** 056625
[22] Kasagi T, Tsutaoka T and Hatakeyama K 2006 *Appl. Phys. Lett.* **88** 172502
[23] Liao S 1998 *Ferromagnetics* (Beijing: Science) p 65